10.

Defaults and Non-Monotonic Reasoning

Outline

- Need for Defaults and Non Monotonic Reasoning
- Classical approaches for dealing with defaults
  - Closed World Reasoning
  - Circumscription
  - Default logic
  - Auto epistemic logic
    - Modal logic
- Modern approaches for dealing with defaults
  - Prioritized rules
  - Argumentation theories
- Example uses of defaults
  - Answering Physics questions
  - Using defaults for credit card authorization

Strictness of FOL

To reason from $P(a)$ to $Q(a)$, need either

- facts about a itself
- universals, e.g. $\forall x(P(x) \supset Q(x))$
  - something that applies to all instances
  - all or nothing!

But most of what we learn about the world is in terms of <u>generics</u>

e.g., encyclopedia entries for ferris wheels, violins, turtles, wildflowers

Properties are not strict for all instances, because

- genetic / manufacturing varieties
  - early ferris wheels
- cases in exceptional circumstances
  - dried wildflowers
- borderline cases
  - toy violins
- imagined cases
  - flying turtles

etc.

Generics vs. universals

✓ Violins have four strings.

vs.

✗ All violins have four strings.

vs.

? All violins that are not E_1 or E_2 or ... have four strings.

(exceptions usually cannot be enumerated)

Similarly, for general properties of individuals

- Alexander the great: ruthlessness
- Ecuador: exports
- pneumonia: treatment

Goal: be able to say a P is a Q <u>in general</u>, but not necessarily

It is reasonable to conclude $Q(a)$ given $P(a)$, unless there is a good reason not to

Here: qualitative version (no numbers)

Varieties of defaults (I)

General statements

- prototypical: The prototypical P is a Q.

  Owls hunt at night.

- normal: Under typical circumstances, P's are Q's.

  People work close to where they live.

- statistical: Most P's are Q's.

  The people in the waiting room are growing impatient.

Lack of information to the contrary

- group confidence: All known P's are Q's.

  Natural languages are easy for children to learn.

- familiarity: If a P was not a Q, you would know it.
  - an older brother
  - very unusual individual, situation or event

Varieties of defaults (II)

Conventional

- conversational: Unless I tell you otherwise, a P is a Q

  "There is a gas station two blocks east."
  the default: the gas station is open.

- representational: Unless otherwise indicated, a P is a Q

  the speed limit in a city

Persistence

- inertia: A P is a Q if it used to be a Q.
  - colours of objects
  - locations of parked cars (for a while!)

Here: we will use "Birds fly" as a typical default.

Non-Monotonic Reasoning

- Ordinary entailment is monotonic

  If KB $\models \alpha$, then KB* $\models \alpha$, for any KB $\subseteq$ KB*

- Default reasoning is non-monotonic

  Can have KB $\models_c \alpha$, KB $\subseteq$ KB', but KB' $\not\models_c \alpha$

  e.g. $\{p\} \models_c \neg q$, but $\{p, q\} \not\models_c \neg q$

Suggests study of <u>non-monotonic reasoning</u>

- start with explicit beliefs
- generate implicit beliefs non-monotonically, taking *defaults* into account
- implicit beliefs may not be uniquely determined (vs. monotonic case)

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Closed-world Reasoning

Reiter's observation:

There are usually many more -ve facts than +ve facts!

Example: airline flight guide provides

DirectConnect(cleveland,toronto) DirectConnect(toronto,northBay)
DirectConnect(toronto,winnipeg) ...

but not: $\neg$DirectConnect(cleveland,northBay)

Conversational default, calle(Closed World Assumption

only +ve facts will be given, relative to some vocabulary

But note: KB $\not\models$ -ve facts (would have to answer: "I don't know")

Proposal: a new version of entailment: KB $\models_c \alpha$ iff KB $\cup$ *Negs* $\models \alpha$

where *Negs* = $\{\neg p \mid p$ atomic and KB $\not\models p\}$

Note: relation to negation as failure

a common pattern:
KB´ = KB $\cup\ \Delta$

Gives: KB $\models_c$ +ve facts and -ve facts

Consistency of CWA

If KB is a set of atoms, then KB $\cup$ *Negs* is always consistent

Also works if KB has conjunctions and if KB has only negative disjunctions

If KB contains $(\neg p \vee \neg q)$. Add both $\neg p, \neg q$.

Problem when KB $\models (\alpha \vee \beta)$, but KB $\not\models \alpha$ and KB $\not\models \beta$

e.g. KB = $(p \vee q)$ *Negs* = $\{\neg p, \neg q\}$

KB $\cup$ *Negs* is inconsistent and so for every α, KB $\models_c \alpha$!

Solution: only apply CWA to atoms that are "uncontroversial"

One approach: <u>GCWA</u>

Negs = $\{\neg p \mid$ If KB $\models (p \vee q_1 \vee \ldots \vee q_n)$ then KB $\models (q_1 \vee \ldots \vee q_n)$ $\}$

When KB is consistent, get:

- KB $\cup$ *Negs* consistent
- everything derivable is also derivable by CWA

Quantifiers and equality

So far, results do not extend to wffs with quantifiers

can have KB $\not\models_c \forall x.\alpha$ and KB $\not\models_c \neg\forall x.\alpha$

e.g. just because for every t, we have KB $\models_c \neg\text{DirectConnect}(\text{myHome}, t)$
does not mean that KB $\models_c \forall x[\neg\text{DirectConnect}(\text{myHome}, x)]$

But may want to treat KB as providing complete information about what individuals exist

Define: KB $\models_{cd}$ α iff KB $\cup$ *Negs* $\cup$ *Dc* $\models \alpha$

where the c_i are all the constants appearing in KB (assumed finite)

where *Dc* is <u>domain closure</u>: $\forall x[x{=}c_1 \vee \ldots \vee x{=}c_n]$,

Get: KB $\models_{cd} \exists x.\alpha$ iff KB $\models_{cd} \alpha[x/c]$, for some c appearing in the KB
KB $\models_{cd} \forall x.\alpha$ iff KB $\models_{cd} \alpha[x/c]$, for all c appearing in the KB

Then add: *Un* is <u>unique names</u>: $(c_i \neq c_j)$, for $i \neq j$

Get: KB $\models_{cdu} (c = d)$ iff c and d are the same constant

→ full recursive query evaluation

Limitation of Closed World Reasoning

- Arbitrary atomic sentences are taken to be false by default
  - We define $|=_c$ as the entailment of KB^+ which is KB augmented with a set of negative literals
  - Because of the negative literals, we end up looking at the models of the KB where the extension of predicates is made as small as possible
  - Two approaches to address the problem
    - Consider forms of entailment where the extension of certain predicates (perhaps not all) is made as small as possible
      - Circumscription
    - Instead of adding to a KB all negative literals that are consistent with the KB, we provide a mechanism for specifying which literals should be added to the KB when it is consistent to do so
      - Default logic

Circumscription

- Introduce an abnormality predicate Ab to talk about the exceptional or abnormal cases where the default should not apply

$$\forall x[\text{Bird}(x) \wedge \neg\text{Ab}(x) \supset \text{Flies}(x)]$$

- Suppose, we have the following facts in the KB:

  Bird (chilly), Bird (tweety), (tweety ≠chilly), ¬Flies(chilly)

Would like to conclude by default Flies(tweety), but KB $|\neq \text{Flies(tweety)}$

because there is an interpretation $\mathfrak{I}$ where $I[\text{tweety}] \in I\,[\text{Ab}]$

Solution: consider only interpretations where $I[\text{Ab}]$ is as small as possible, relative to KB

for example: KB requires that $I[\text{chilly}] \in I[\text{Ab}]$

this is sometimes called "circumscription" since we circumscribe the Ab predicate

Generalizes to many Ab_i predicates

Default Logic

- A KB is considered as *default theory* consisting of two parts
  - a set F of first-order sentences as usual
  - A set D of default rules which specify what assumptions can be made and when
- Default logic specifies set of implicit beliefs incorporating facts in F and incorporating as many default assumptions as we can given D
  - Some times there can be more than one set of candidate assumptions

Default Rules

Default logic KB uses two components: KB = $\langle F, D \rangle$

- F is a set of sentences (facts)
- D is a set of <u>default rules</u>: triples $\langle \alpha : \beta \,/\, \gamma \rangle$ read as

If you can infer α, and β is *consistent*, then infer γ

α: the prerequisite, β: the justification, γ: the conclusion

e.g. $\langle \mathrm{Bird(tweety)} : \mathrm{Flies(tweety)} \;/\; \mathrm{Flies(tweety)} \rangle$

treat $\langle \mathrm{Bird}(x) : \mathrm{Flies}(x) \,/\, \mathrm{Flies}(x) \rangle$ as <u>set</u> of rules

Default rules where $\beta = \gamma$ are called <u>normal</u> and write as $\langle \alpha \Rightarrow \beta \rangle$

will see later a reason for wanting non-normal ones

A set of sentences E is an <u>extension</u> of $\langle F, D \rangle$ iff for every sentence π, E satisfies the following:

$$\pi \in E \text{ iff } F \cup \Delta \models \pi, \quad \text{where } \Delta = \{\gamma \mid \langle \alpha : \beta \,/\, \gamma \rangle \in D,\ \alpha \in E, \neg\beta \notin E\}$$

So, an extension E is the set of entailments of $F \cup \{\gamma\}$, where the γ are assumptions from D.

to check if E is an extension, guess at Δ and show that it satisfies the above constraint

Limitations of Default Logic

- In default logic, even though we can reason with defaults, we cannot reason about them

$$\langle \alpha : \beta / \gamma \rangle \rightarrow \langle \alpha : \beta / (\gamma \vee \delta) \rangle$$

- Ie, there is no notion of entailment amongst defaults
- In contrast, in circumscription
  - Defaults are ordinary sentences in the language
  - Forces us to handle defaults in terms of abnormalities as opposed to directly representing them as in default logic
- Solution: Auto epistemic logic
  - Defaults are represented as sentences and can be reasoned with

Autoepistemic Logic

Solution: express defaults as *sentences* in an extended language that talks about belief explicitly

for any sentence α, we have another sentence $\mathbf{B}\alpha$

$\mathbf{B}\alpha$ says "I believe α": autoepistemic logic

e.g. $\forall x[\mathrm{Bird}(x) \wedge \neg\mathbf{B}\neg\mathrm{Flies}(x) \supset \mathrm{Flies}(x)]$

All birds fly except those that I believe to not fly =

Any bird not believed to be flightless flies.

No longer expressing defaults using formulas of FOL.

Semantics of belief

These are not sentences of FOL, so what semantics and entailment?

- modal logic of belief provide semantics
- for here: treat $\mathbf{B}\alpha$ as if it were an new atomic wff
- still get entailment: $\forall x[\mathrm{Bird}(x) \wedge \neg\mathbf{B}\neg\mathrm{Flies}(x) \supset \mathrm{Flies}(x) \vee \mathrm{Run}(x)]$

Main property for set of implicit beliefs, E:

1. If $E \models \alpha$ then $\alpha \in E$. (closed under entailment)
2. If $\alpha \in E$ then $\mathbf{B}\alpha \in E$. (positive introspection)
3. If $\alpha \notin E$ then $\neg\mathbf{B}\alpha \in E$. (negative introspection)

Any such set of sentences is called stable

Note: if E contains p but does not contain q, it will contain $\mathbf{B}p$, $\mathbf{BB}p$, $\mathbf{BBB}p$, $\neg\mathbf{B}q$, $\mathbf{B}\neg\mathbf{B}q$, $\mathbf{B}(\mathbf{B}p \wedge \neg\mathbf{B}q)$, *etc.*

(See modal logic handout for more information on modal logic)

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Prioritized Defaults

- Two basic ideas
  - Conflicts
    - Specify which two literals compete in the sense that cannot be true together
    - For example:

      opposes(discount(?product,“5%”), discount(?product,“10%”))
  - Override
    - Each rule has a label
    - Label can be used to say that one rule overrides another
    - For example:

      overrides(rule1, rule2)   means rule1 is higher-priority than rule2

Example 1

Rule labels

```
@d1     flies(X) :- bird(X).
@d2     neg flies(X) :- penguin(X).
        bird(tweety).
        penguin(tweety).
        overrides(d2,d1).
```

Explicit negation

Rule d2 has higher priority than rule d1

Answer: { *bird(tweety), penguin(tweety), neg flies(tweety)* }

Example adapted from Hui Wan

Example

- Vendor’s rules that prescribe how buyer must place or modify an order:
  A. 14 days ahead if the buyer is a qualified customer
  B. 30 days ahead if the ordered item is a minor part
  C. 2 days ahead if the ordered item’s item-type is backlogged at the vendor, the order is a modification to reduce the quantity of the item, and the buyer is a qualified customer
  D. 45 days ahead if the buyer is a walk-in customer

- Multiple rules can apply
- Rule ordering may be partial

Example Adapted from Benjamin Grosof

Example

- @prefCust orderModifNotice(?Order,14days)
  $\leftarrow$ preferredCustomerOf(?Buyer,SupplierCo) $\wedge$
  purchaseOrder(?Order,?Buyer,SellerCo) ;
- @smallStuff orderModifNotice(?Order,30days)
  $\leftarrow$ minorPart(?Buyer,?Seller,?Order) $\wedge$
  purchaseOrder(?Order,?Buyer,SupplierCo) ;
- @reduceTight orderModifNotice(?Order,2days)
  $\leftarrow$ preferredCustomerOf(?Buyer,SupplierCo) $\wedge$
  orderModifType(?Order,reduce) $\wedge$
  orderItemIsInBacklog(?Order) $\wedge$
  purchaseOrder(?Order,?Buyer,SupplierCo) ;
- overrides(reduceTight , prefCust) ;
- opposes(orderModifNotice(?Order,?X), orderModifNotice(?Order,?Y)) :- ?X $\neq$?Y ;

- NB: Rule D, and prioritization about it, were omitted above for sake of brevity.

Motivation for Arguments Theories

$@a$ $p.$
$@b$ $q.$
$@c$ $s.$
opposes(p, q).
opposes(q, s).
overrides(a, b).
overrides(b, c).

p and *q* cannot be true together. Ditto for *q* and *s*.

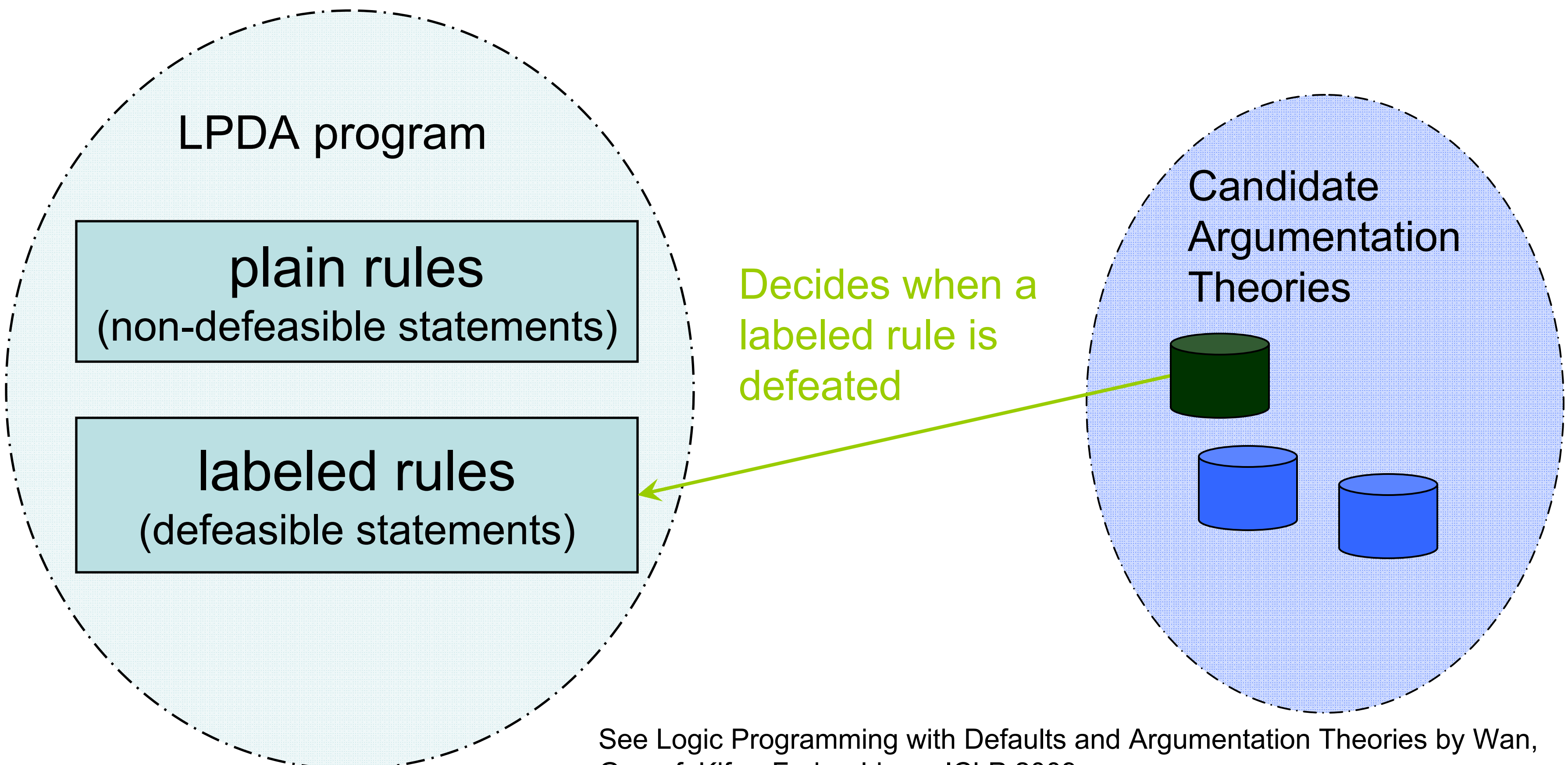
Answer 1: $\{p, not\ q, not\ s\}$
Intuition: rule b is defeated by rule a, rule c is defeated by rule b.

Answer 2: $\{p, not\ q, s\}$
Intuition: rule b is defeated by rule a, so rule c is not defeated.

No single intuition (argumentation) works for all application domains.

Example adapted from Hui Wan

Logic Programs with Defaults and Argumentation Theory



See Logic Programming with Defaults and Argumentation Theories by Wan, Grosof, Kifer, Fodor, Liang, ICLP 2009

Answering Physics Questions

How much force is required to lift a 50-newton object with an acceleration of 10 m/s2?

(A)10 N
(B)50 N
(C)100 N
(D)150 N
(E)200 N

The answer depends on what value of g is used in computing the answer.

By default we assume that an object is on earth unless stated otherwise.

Answering Physics Questions

- Using Abnormality Predicate
  - Assume that an object is on earth unless stated otherwise

    ```
    ?obj[location->Earth] :- ?obj:Object, not ab(?obj).
    ```

  - If an object is located on a non-earth object, then it is "ab" with respect to the "All objects are on Earth" assumption.

    ```
    ab(?obj) :-
      ?obj:Object[location->?loc],
      ?loc:HeavenlyBody,
      not ?loc :=: Earth.
    ```

Answering Physics Questions

- Using Overrides
  - For any object conclude that it is on earth

    @{Rule1}?obj[location->Earth] :- ?obj:Object.
  - If an object is on some planet (or more generally, some heavenly body) that is not Earth, the conclude that that object is not on Earth

    @{Rule2}neg ?object[location->Earth] :- ?object:Object[location->?loc],
    ?loc:HeavenlyBody, not ?loc :=: Earth.
  - Rule2 overrides Rule 1

    _overrides(Rule2, Rule1).

Using Overrides for Credit Card Authorization

- See the handout for a life sized example for how the priorities can be used in the context of credit card authorization

Readings

- Required Readings
  - For classical techniques for defaults and non monotonic reasoning
    - Chapter 11 of Brachman & Levesque textbook
  - Modal Logic
    - For an introduction, refer to the modal logic handout extracted from the textbook: Multi-Agent Systems, by Yoav Shoham & Kevin Leyton-Brown
- Optional Readings
  - For a more in-depth understanding, refer to either of the following
    - Truth and Modality in Knowledge Representation by Raymond Turner
    - A New Introduction to Modal Logic by Hughes and Creswell