8a.
Reasoning with Horn Clauses

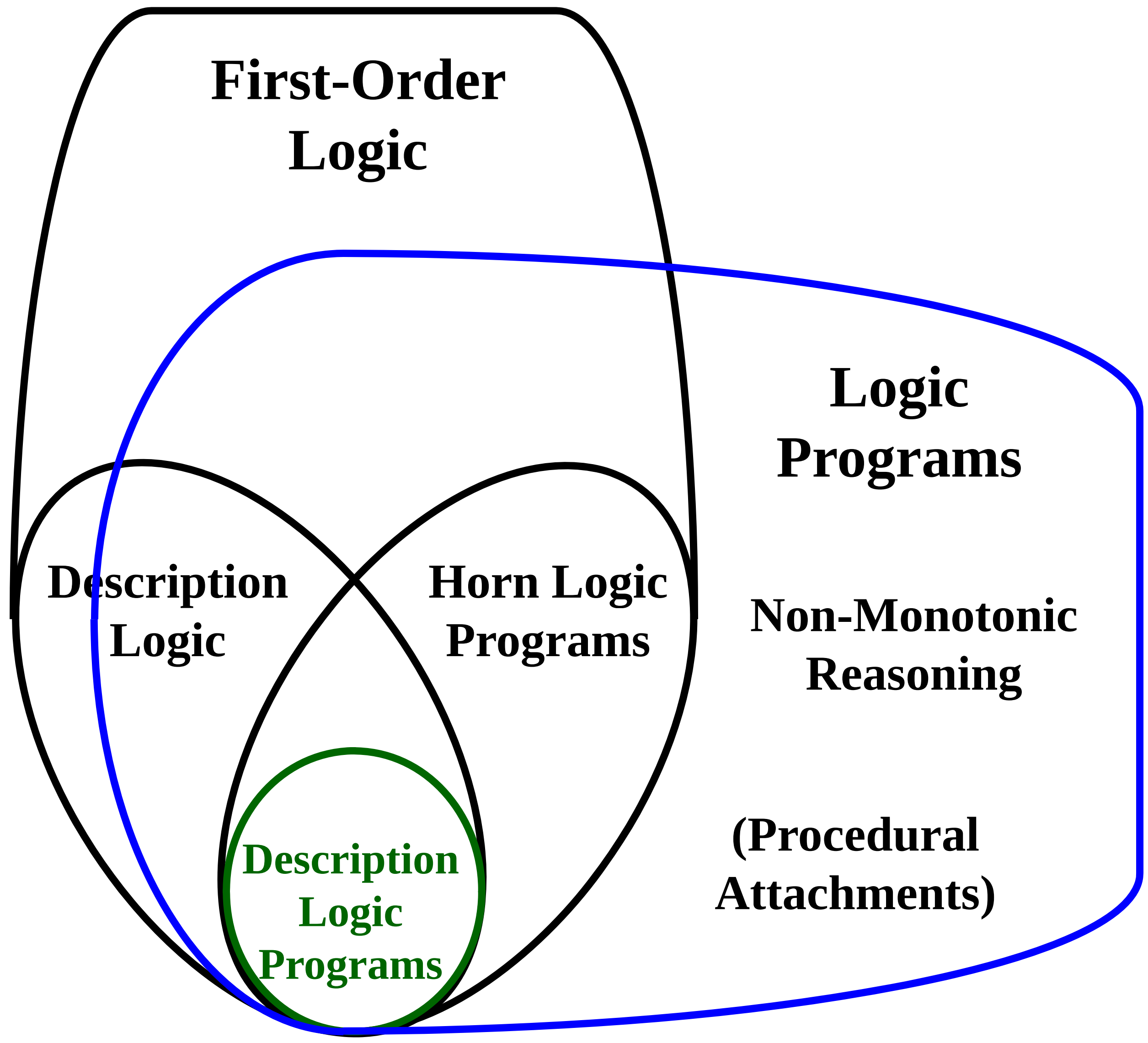
Review

- Lecture 1: What is KR&R
  - KR Hypothesis: Explicit representation of knowledge provides propositional account and causal explanation for intelligent behavior
- Lecture 2: Object-Oriented Representation
  - Frames provide a way to organize knowledge
- Lecture 3-5: Structured Descriptions
  - Adding structure to the definition of objects; sound, complete and efficient reasoning
- Lecture 6: Ontologies
  - Engineering discipline of deciding which class, function and relation symbols to use in representing a domain
- Lecture 7: Knowledge Representation in Social Context
  - KR&R concepts for the Web

Next Four Lectures

- Frames and structured descriptions provide useful subsets of FOL
  - Their expressive power, however, is limited
- In lectures 8 through 11, we will study more expressive representations
  - Reasoning with Horn Clauses
    - Foundation for logic programming family of languages
  - Procedural control of reasoning
    - Negation as Failure - a practical alternative to classical negation
  - Production Systems
    - Foundation of expert systems / rule-based systems
  - Advanced logics
    - Combining rules with object-oriented and structured representations, higher order logic, modal logic
  - Non Monotonic Reasoning
    - Representing default knowledge, answer set programming

Expressive Overlaps among KRs



Reasoning with Horn Clauses

- Definitions
- SLD Resolution
- Forward and Backward Chaining
- Efficiency of reasoning with Horn Clauses
- Horn FOL vs Horn LP

Definitions

- Term
- Formula
- Atomic Formula
- Sentence
- Literal
- Clause

Definitions

- Term
  - The set of terms of FOL is the least set satisfying these conditions:
    - every variable is a term
    - if tl . . . . . tn are terms, and f is a function symbol of arity n, then f(tl . . . . . tn) is a term
- Formula
  - The set of formulas of FOL is the least set satisfying these constraints:
    - if tl . . . . . tn are terms, and P is a predicate symbol of arity n, then P(t1 . . . . . tn) is a formula;
    - if t1 and t2 are terms, then tl=t2 is a formula;
    - if α and β are formulas, and x is a variable, then ¬α, α \/ β, α /\ β, □x α, and Exists α, are formulas.
- Atomic Formula
  - Formulas of first two types above
- Sentence
  - Any formula with no free variables
- Literal
  - Atomic formula or its negation
- Clause
  - A finite set of literals

Resolution

For the premises (p => q) and (q => r), we want to prove (p => r)

1. $\{\neg p, q\}$	Premise
2. $\{\neg q, r\}$	Premise
3. $\{p\}$	Negated Goal
4. $\{\neg r\}$	Negated Goal
5. $\{q\}$	3, 1
6. $\{4\}$	5, 2
7. $\{\}$	6, 4

Horn clauses

Clauses are used two ways:

- as disjunctions: (rain $\vee$ sleet)
- as implications: ($\neg$child $\vee$ $\neg$male $\vee$ boy)

Here focus on 2nd use

Horn clause = at most one +ve literal in clause

- positive / definite clause = exactly one +ve literal

  e.g. $[\neg p_1, \neg p_2, \ldots, \neg p_n, q]$
- negative clause = no +ve literals (also, referred to as integrity constraints)

  e.g. $[\neg p_1, \neg p_2, \ldots, \neg p_n]$ and also [ ]

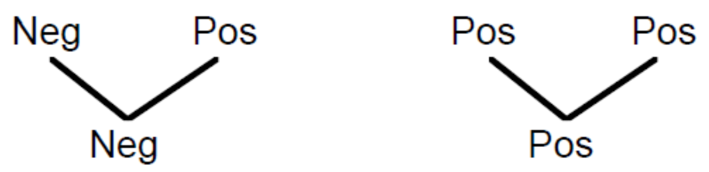
Note: $[\neg p_1, \neg p_2, \ldots, \neg p_n, q]$ is a representation for
$(\neg p_1 \vee \neg p_2 \vee \ldots \vee \neg p_n \vee q)$ or $[(p_1 \wedge p_2 \wedge \ldots \wedge p_n) \supset q]$

so can read as: If p_1 and p_2 and ... and p_n then q

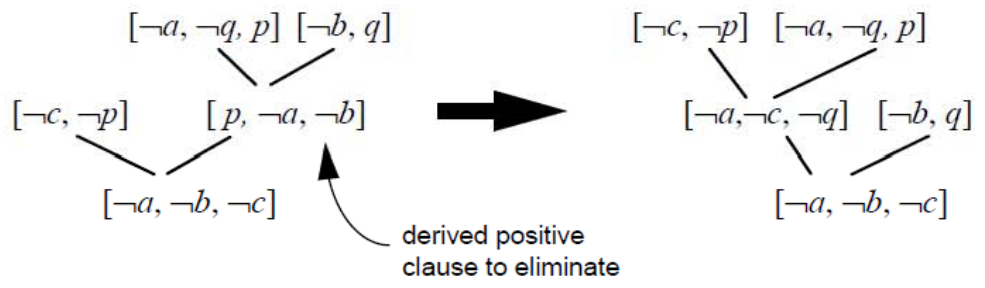
and write as: $p_1 \wedge p_2 \wedge \ldots \wedge p_n \Rightarrow q$ or $q \Leftarrow p_1 \wedge p_2 \wedge \ldots \wedge p_n$

Resolution with Horn clauses

Only two possibilities:



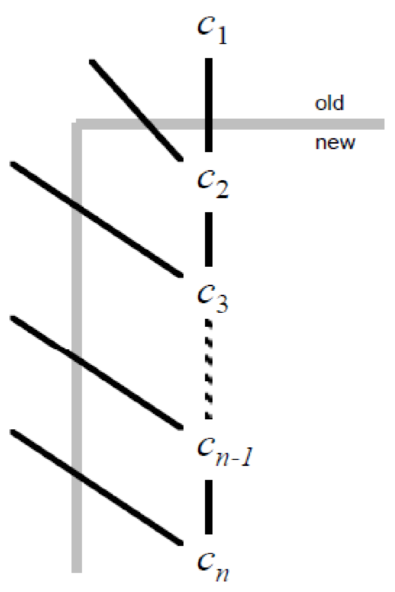
It is possible to rearrange derivations of negative clauses so that all new derived clauses are negative



Further restricting resolution

Can also change derivations such that each derived clause is a resolvent of the previous derived one (negative) and some positive clause in the original set of clauses

- Since each derived clause is negative, one parent must be positive (and so from original set) and one parent must be negative.
- Chain backwards from the final negative clause until both parents are from the original set of clauses
- Eliminate all other clauses not on this direct path

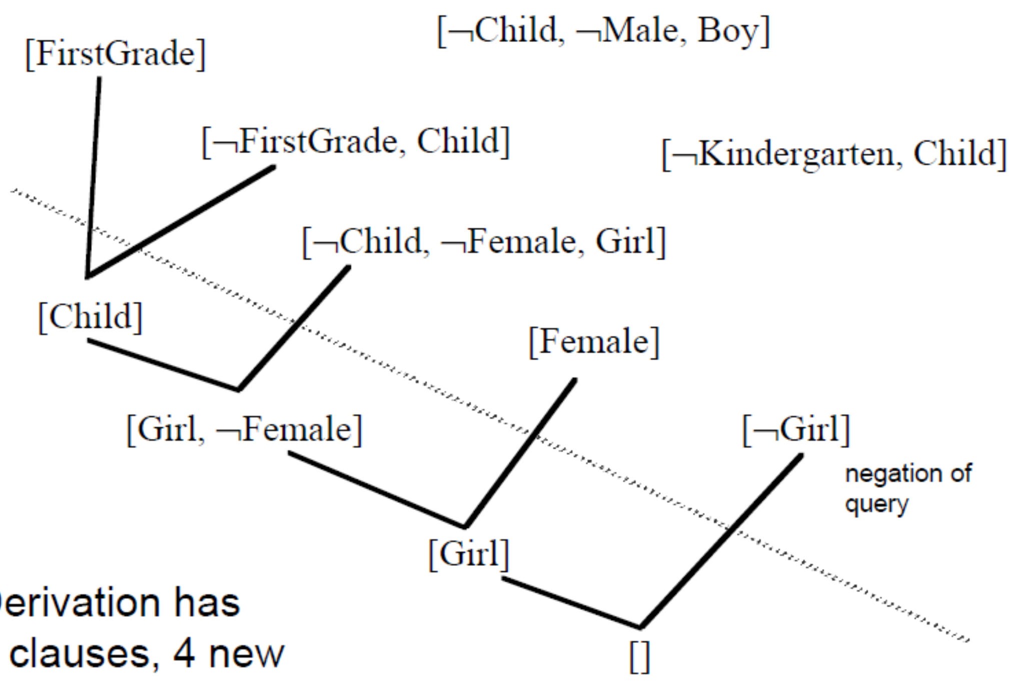


Example 1

KB

FirstGrade
FirstGrade ⊃ Child
Child ∧ Male ⊃ Boy
Kindergarten ⊃ Child
Child ∧ Female ⊃ Girl
Female

Show that KB |= Girl



Derivation has
9 clauses, 4 new

SLD version of Example 1

KB

FirstGrade

FirstGrade $\supset$ Child

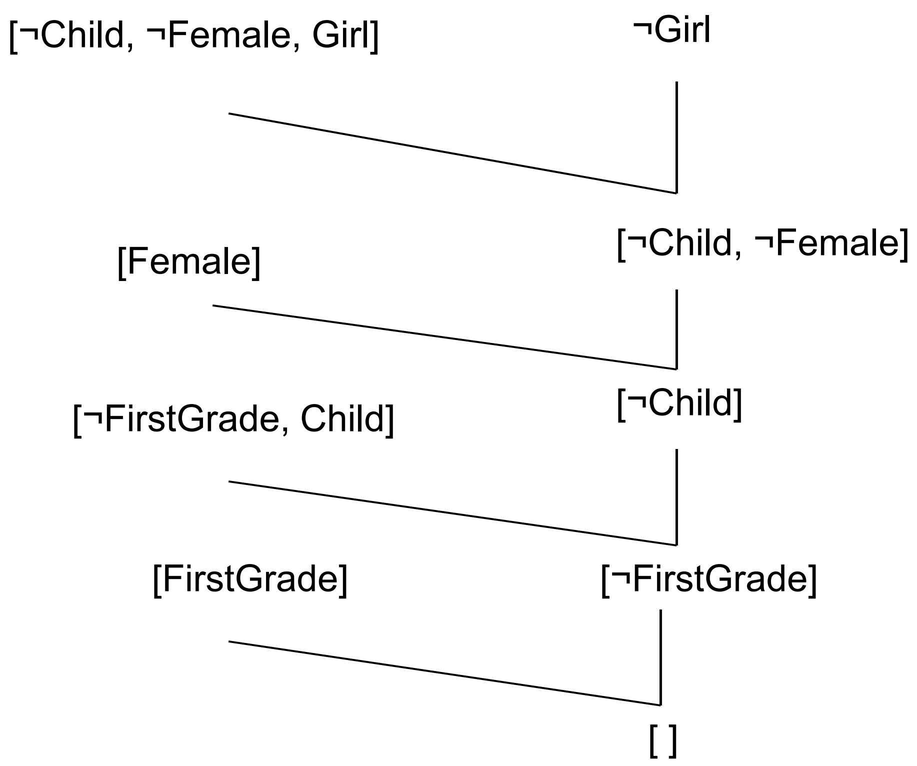
Child $\wedge$ Male $\supset$ Boy

Kindergarten $\supset$ Child

Child $\wedge$ Female $\supset$ Girl

Female

Show that KB |= Girl



SLD Resolution

An <u>SLD-derivation</u> of a clause c from a set of clauses S is a sequence of clause $c_1, c_2, \ldots c_n$ such that $c_n = c$, and

1. $c_1 \in S$
2. c_{i+1} is a resolvent of c_i and a clause in S

Write: $S \overset{\text{SLD}}{\rightarrow} c$

SLD means S(elected) literals
L(inear) form
D(efinite) clauses

Note: SLD derivation is just a special form of derivation and where we leave out the elements of S (except c_1)

In general, cannot restrict ourselves to just using SLD-Resolution

Proof: $S = \{[p, q], [p, \neg q], [\neg p, q]\ [\neg p, \neg q]\}$. Then $S \rightarrow []$.

Need to resolve some $[\,\rho\,]$ and $[\,\overline{\rho}\,]$ to get [].
But S does not contain any unit clauses.

So will need to derive both $[\,\rho\,]$ and $[\,\overline{\rho}\,]$ and then resolve them together.

Completeness of SLD

However, for Horn clauses, we can restrict ourselves to SLD-Resolution

Theorem: SLD-Resolution is refutation complete for Horn clauses: $H \rightarrow []$ iff $H \overset{\text{SLD}}{\rightarrow} []$

So: H is unsatisfiable iff $H \overset{\text{SLD}}{\rightarrow} []$

This will considerably simplify the search for derivations

Note: in Horn version of SLD-Resolution, each clause in the $c_1, c_2, \ldots, c_n$, will be negative

So clauses H must contain at least one negative clause, c_1 and this will be the only negative clause of H used.

Typical case:

- KB is a collection of positive Horn clauses
- Negation of query is the negative clause

Example 1 (again)

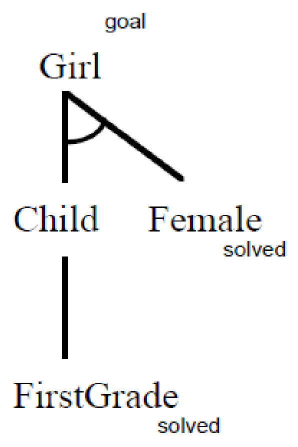
KB

FirstGrade
FirstGrade $\supset$ Child
Child $\wedge$ Male $\supset$ Boy
Kindergarten $\supset$ Child
Child $\wedge$ Female $\supset$ Girl
Female

SLD derivation

$$\begin{tikzcd}[\neg\text{Girl}] \arrow[d, dash] \\ [\neg\text{Child}, \neg\text{Female}] \arrow[d, dash] \\ [\neg\text{Child}] \arrow[d, dash] \\ [\neg\text{FirstGrade}] \arrow[d, dash] \\ []\end{tikzcd}$$

alternate representation



A goal tree whose nodes are atoms, whose root is the atom to prove, and whose leaves are in the KB

Show KB $\cup$ $\{\neg\text{Girl}\}$ unsatisfiable

Back-chaining procedure

```
Solve[$q_1, q_2, \ldots, q_n$] =          /* to establish conjunction of $q_i$ */
    If $n$=0 then return YES;             /* empty clause detected */
    For each $d \in$ KB do
        If $d$ = [$q_1, \neg p_1, \neg p_2, \ldots, \neg p_m$]     /* match first $q$ */
            and                                                    /* replace $q$ by -ve lits */
          Solve[$p_1, p_2, \ldots, p_m, q_2, \ldots, q_n$]         /* recursively */
        then return YES
    end for;                  /* can't find a clause to eliminate $q$ */
    Return NO
```

Depth-first, left-right, back-chaining

- depth-first because attempt p_i before trying q_i
- left-right because try q_i in order, 1,2, 3, ...
- back-chaining because search from goal q to facts in KB p

This is the execution strategy of Prolog

First-order case requires unification *etc.*

Problems with back-chaining

Can go into infinite loop

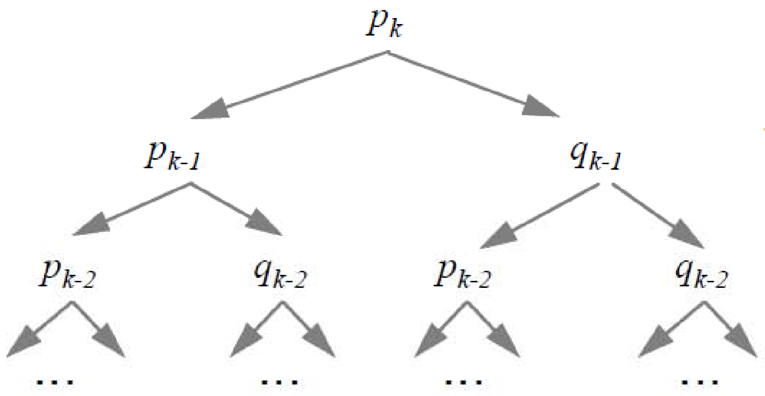
tautologous clause: $[p\ ,\ \neg p]$ (corresponds to Prolog program with `p :- p`).

Previous back-chaining algorithm is inefficient

Example: Consider $2n$ atoms, $p_0, \ldots, p_{n-1}, q_0, \ldots, q_{n-1}$ and $4n$-4 clauses

$$(p_{i-1} \Rightarrow p_i),\ (q_{i-1} \Rightarrow p_i),\ (p_{i-1} \Rightarrow q_i),\ (q_{i-1} \Rightarrow q_i).$$

With goal p_k the execution tree is like this



Solve[p_k] eventually fails after 2^k steps!

Is this problem inherent in Horn clauses?

Forward-chaining

Simple procedure to determine if Horn KB $\models q$.

main idea: mark atoms as solved

```
1. If $q$ is marked as solved, then return YES
2. Is there a $\{p_1, \neg p_2, \ldots, \neg p_n\} \in$ KB such that
   $p_2, \ldots, p_n$ are marked as solved, but the
   positive lit $p_1$ is not marked as solved?
       no:    return NO
       yes:   mark $p_1$ as solved, and go to 1.
```

FirstGrade example:

Marks: FirstGrade, Child, Female, Girl then done!

Note: FirstGrade gets marked since all the negative atoms in the clause (none) are marked

Observe:

- only letters in KB can be marked, so at most a linear number of iterations
- not goal-directed, so not always desirable
- a similar procedure with better data structures will run in *linear* time overall

First-order undecidability

Even with just Horn clauses, in the first-order case we still have the possibility of generating an infinite branch of resolvents.

KB:

$\text{LessThan}(\text{succ}(x),y) \Rightarrow \text{LessThan}(x,y)$

Query:

$\text{LessThan}(\text{zero},\text{zero})$

As with full Resolution, there is no way to detect when this will happen

$$\begin{tikzcd}
{[\neg \text{LessThan}(0,0)]} \arrow[d, "x/0,\ y/0"] \\
{[\neg \text{LessThan}(1,0)]} \arrow[d, "x/1,\ y/0"] \\
{[\neg \text{LessThan}(2,0)]} \arrow[d, "x/2,\ y/0"] \\
\cdots
\end{tikzcd}$$

There is no procedure that will test for the satisfiability of first-order Horn clauses

the question is undecidable

As with non-Horn clauses, the best that we can do is to give control of the deduction to the *user*

to some extent this is what is done in Prolog, but we will see more in "Procedural Control"

Horn FOL vs Horn LP

- In Horn LP, the conclusions are limited to ground atomic formulas. For example:
  - Suppose, we have[1]:

    DangerousTo(?x,?y) $\leftarrow$ PredatorAnimal(?x) $\wedge$ Human(?y);

    PredatorAnimal(?x) $\leftarrow$ Lion(?x)

    Lion(Simba)

    Human(Joey)
  - In Horn LP, we can derive
    - I1 = {Lion(Simba), Human(Joey)}
    - I2 = {PredatorAnimal(Simba),Lion(Simba), Human(Joey)}
    - I3 = {DangerousTo(Simba,Joey), PredatorAnimal(Simba),Lion(Simba), Human(Joey)}
  - In Horn FOL, we will also derive:
    - DangerousTo(Simba,?y) $\Leftarrow$ Human(?y)
    - $\neg$Human(?y) $\Leftarrow$ $\neg$DangerousTo(Simba,?y).
- Horn LP is the foundation of logic programming and Prolog

1. Example adapted from Grosof, Kifer & Dean

Recommended Reading

- Chapter 5 of Brachman & Levesque textbook